Short Papers

Optimum Termination Networks for Tightly Coupled Microstrip Lines Under Random and Deterministic Excitations

Smain Amari and Jens Bornemann

***Abstract*—A general method to determine the termination network of multiple coupled lines for lowest input return loss is presented. Each line is connected to the ground and its nearest neighbors by resistors whose values are determined by minimizing the reflected power. It is demonstrated that under these conditions the optimum termination network depends on the excitation. The inequality of the modal propagation constants requires that the length of the lines be properly taken into account when designing a termination network which ensures maximum power delivery to the loads. Matching networks for five and seven coupled transmission lines under different excitations are presented. For the more practical case of unknown excitations, a design procedure based on minimizing the reflected power for independent random-incident voltage variables is presented.**

***Index Terms*—Coupled transmission lines, microstrip circuits, termination network.**

I. Introduction

Systems of coupled lines play a major role in modern microwave integrated circuits. They are used in microwave couplers and filters or simply as a means of channeling electromagnetic energy between different parts of a microwave circuit. Proper design of the transmission section and loads is essential to any effective exchange of energy and information.

Maximum power delivery through a section of a transmission line requires proper matching of the load to the transmission line. Ideally, a termination network consisting of $N(N+1)/2$ resistors, such that the load admittance $[Y_L]$ is equal to the characteristic admittance matrix $[Y_c]$ of the coupled lines, ensures zero reflection [1]. However, due to practical limitations, it is not always possible to terminate the lines in their characteristic admittance matrix, as this leads to a matrix-like termination network interconnecting all lines and ground with each other, thus requiring three-dimensional (3-D) installations of load resistors. Instead, a more practical termination network has been suggested in [2] where each line is resistively connected to ground and only to its nearest neighbors (see Fig. 1).

However, with respect to the design of such a network, there are several options for calculating the individual load resistances.

1) *Minimizing the Sum of the Magnitudes of the Entries of the Voltage Reflection Matrix* $[\rho_V]$: The resistances are then obtained by minimizing the positive definite quantity [2]

$$R = \sum_{j=1}^{j=N} \sum_{i=1}^{i=N} |[\rho_V]_{ij}|^2. \tag{1}$$

However, by eliminating the information contained in the phases of the reflection matrix, this procedure does not take advantage of the destructive interferences that take place. It is conceivable that very small total reflections can result from elementary reflections, which are themselves large, but interfering destructively.

2) *Minimizing the Sum of the Squares of the Total Reflected Voltages from the Loads*: In this case, a modified version of (1) is minimized as follows:

$$R' = \sum_{i=1}^{i=N} \left| \sum_{j=1}^{j=N} [\rho_V]_{ij} \right|^2. \tag{2}$$

Equation (2) now involves the magnitudes of the total reflected voltages on each line. Therefore, we are including—to our advantage—the phases of the individual reflected voltages and their destructive interference. Moreover, we are including the finite length of the transmission-line section, thus taking into account mode coupling between the lines. [Other criteria, such as minimizing the reflected voltage on a given line only, lead to expressions similar to (2).]

3) *Minimizing the Total Reflected Power from the Loads*: This is the approach we are proposing in this paper. The reader might feel that this is an immediate disadvantage, since well-matched termination networks are known to be *excitation dependent*. However, we counteract this objection by assuming—in Section II-B—an *arbitrary distribution* of exciting waves in order to find the optimum termination network.

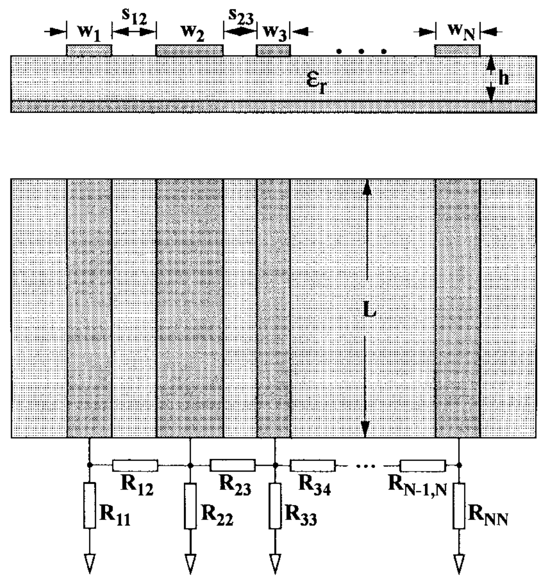

Fig. 1. System of coupled microstrip lines with its termination network.

II. Theory

A. Deterministic Excitation

Fig. 1 shows a system of coupled lines of length L along with a realistic termination network in which the ends of the lines are connected to the ground and only to their immediate neighbors according to [2].

Manuscript received November 10, 1995; revised June 20, 1997.
The authors are with the Department of Electrical and Computer Engineering, Laboratory for Lightwave Electronics, Microwaves and Communications, LLiMiC, University of Victoria, Victoria B.C., Canada V8W 3P6.
Publisher Item Identifier S 0018-9480(97)07113-5.

In microstrip transmission-line design, it is usually assumed that the structure is excited with a frequency low enough so that only the fundamental transmission-line modes are propagating. Then the voltages and currents on the lines can be expanded in terms of the eigencurrent and eigenvoltage matrices $[M_I]$, and $[M_V]$ [2]–[4]:

$$[I(z)] = [M_I][Q^+][K^+] - [M_I][Q^-][K^-] \tag{3}$$

$$[V(z)] = [M_V][Q^+][K^+] + [M_V][Q^-][K^-] \tag{4}$$

where $[Q^{\pm}] = \mathrm{diag}\,[\exp(\pm j\beta_k z)]$. The propagation constants β_k of an N-line coupled-line system ($k = 1 \cdots N$) as well as the matrices $[M_V]$ and $[M_I]$ are obtained from the spectral-domain solution of the two-dimensional (2-D) multiple coupled-line problem, e.g., [4]–[6]. $[K^{\pm}]$ are expansion coefficients used in the spectral-domain approach. The incident and reflected voltage waves are $[M_V][Q^+][K^+]$ and $[M_V][Q^-][K^-]$, respectively. Using the boundary conditions at the terminations

$$I(z = 0) - [Y_L][V(z = 0)] = 0 \tag{5}$$

we get the reflected currents and voltages in terms of the incident voltages

$$[V_{\mathrm{ref}}] = [\rho_V][V_{\mathrm{inc}}] \tag{6}$$

$$[I_{\mathrm{ref}}] = [Y_C][\rho_V][V_{\mathrm{inc}}] \tag{7}$$

where

$$[Y_C] = [M_I][M_V]^{-1}. \tag{8}$$

Note that (8) is identical to [1, eq. (8b)]. The reflection coefficient matrix is

$$[\rho_V] = ([Y_C] + [Y_L])^{-1}([Y_C] - [Y_L]). \tag{9}$$

The average power reflected from the load is then

$$P_{\mathrm{ref}} = \tfrac{1}{2}\,\mathrm{Re}\,[V_{\mathrm{inc}}]^T[\rho_V]^T[Y_C][\rho_V][V_{\mathrm{inc}}] \tag{10}$$

where the superscript T denotes the transpose of a matrix.

In the ideal, though nonpractical structure of a matrix-like termination network (see Section I), P_{ref} vanishes since the reflection coefficient matrix $[\rho_V]$ vanishes, and the termination network is independent of the excitation $[V_{\mathrm{inc}}]$. However, in the practical case of Fig. 1, a well-matched termination network—obtained by minimizing (10)—depends on the nature of the incident wave. For example, if only the ith line is excited, then minimizing the reflected power is equivalent to minimizing $([\rho_V]^T[Y_C][\rho_V])_{ii}$ which, in general, depends on i. Therefore, only if the incident voltage wave at the loads is known can a matching termination network, which guarantees maximum power delivery, be designed through (10).

However, in most situations, the excitations are only known at the launching point, i.e., at the generator. Consequently, for transmission-line structures, which support propagating modes other than pure TEM, the propagating wave will have a different distribution when it reaches the loads. Especially in this non-TEM case of tightly coupled microstrip lines, the length of the transmission-line section is of fundamental importance, as large differences between the modal propagation constants are typically observed.

The reflected power from the loads, in terms of the incident voltage at the other end (generator), can be derived similarly to (10) and results in

$$P_{\mathrm{ref}} = \tfrac{1}{2}\,\mathrm{Re}\,[V_{\mathrm{inc}}^g]^T[M_V]^{-1T}[Q^+][M_V]^T[\rho_V]^T \cdot [Y_C][\rho_V][M_V][Q^-][M_V]^{-1}[V_{\mathrm{inc}}^g]. \tag{11}$$

Here, $[V_{\mathrm{inc}}^g]$ is the incident voltage at the generator. Equations (10) and (11) are identical only if the length is reduced to zero or if all propagation constants are identical ($\beta_k = \beta_{\mathrm{TEM}}$). In the latter case, $[Q^+]$ and $[Q^-]$ are equal to the identity matrix multiplied by $e^{\pm j\beta_{\mathrm{TEM}}L}$; their position in (11) is inconsequential, since they commute with any matrix, and by moving $[Q^+]$ to directly multiply $[Q^-]$, and noting that $[Q^+][Q^-]$ is the unit matrix, we get (10). In general, however, the reflected power depends on the length L and the propagation constants β_k, and consequently, requires a different termination network.

III. Arbitrary Excitations

It is not practical to terminate the network with loads which ensure minimum reflection only for a well-specified excitation unless the network is itself dedicated to such a voltage distribution. In general, the network is excited by waves of arbitrary (stochastic) distribution. It is, therefore, important to design a termination network which reflects a minimum power under these conditions.

A simple model of the random distribution of the incident voltages on the lines is assumed. The excitations on the lines are taken as independent random variables with the same distribution. Absent of any other *a priori* information about the nature of such a distribution, we assume a discrete random voltage on each line, which, similar to situations in digital signal transmission, takes on the values $+1$ or -1 V with equal probabilities of 1/2. Note that a more complicated probability density of excitations can be incorporated, but would differ only in the generation of the input voltage vector.

The termination network is now determined by requiring the reflected power averaged over the ensemble be minimum:

$$\overline{P}_{\mathrm{ref}} = \text{minimum}. \tag{12}$$

The notation $\overline{X}$ stands for the average of X over the random variable $[V_{\mathrm{inc}}]$.

In this simple model, the ensemble average in (12) is obtained by adding the weighted contributions from all possible values of the voltages on the lines. Since all the probabilities are taken equal to 1/2, they can be factored out. If a more complicated probability density of the excitations is assumed, (12) should be implemented accordingly.

IV. Results

A. Five Symmetric Lines

We first consider a termination network for five coupled symmetric lines, a structure which was also investigated in [2]. Our results for the propagation constants of the five modes, computed through the spectral-domain immittance approach [6], compare well with those presented in [2]: (2.9300, 2.6603, 2.6375, 2.6363, 2.636 35) against (2.9117, 2.6551, 2.6372, 2.6368, 2.6367). To further check our code, we also computed the characteristic impedance of an isolated microstrip line of width 0.02 mm on a substrate of thickness 0.1 mm and dielectric constant 12.9. At 5 and 50 GHz, we obtain 80.71 and 82.25 Ω, which compare well with 78.9 and 84.1 Ω reported in [2]. The discrepancies between our results and those in [2] are believed to be caused by the number of basis functions. We used two basis functions (with the edge condition) on each line. It was also necessary to sum a large number of terms because of the strong coupling between the lines. However, for the demonstration of this

TABLE I
ENTRIES OF TERMINATION NETWORK [Ω], OF FIVE-LINE GEOMETRY. (SEE [2] FOR LINE PARAMETERS)

	equation (1) R_{ij} independent of excitation	equation (2) R_{ij} independent of excitation	equation (10) random excitation	Elements from characteristic matrix $[Y_C]$
R_{11}	129.05	136.48	125.11	126.24
R_{12}	75.20	80.88	76.26	76.24
R_{22}	319.48	345.12	313.08	368.60
R_{23}	82.14	108.13	80.83	77.81
R_{33}	331.98	288.35	374.90	633.41
R_{34}	82.14	60.77	80.83	77.81
R_{44}	319.48	345.12	313.08	368.60
R_{45}	75.20	126.76	76.26	76.24
R_{55}	129.05	136.48	125.11	126.24
$Pr\%$	0.107	1.56	0.102	0.125

method, the actual values are of secondary importance. The major point here is the fact that the propagation constants are different for different modes.

A variety of strategies to calculate the termination networks were investigated:

1) minimizing the total reflected power;
2) minimizing the sum of the squares of the reflected voltages (2);
3) minimizing the sum of the squares of the entries of the voltage reflection matrix (1).

In the last two cases, the termination network is independent of the excitation; however, this is not the case when the total reflected power is minimized. In all cases, the minimizing procedure is based on a quasi-Newton method and a finite-difference approximation of the gradient.[1]

Fig. 2 shows the total reflected power as a function of $Y_L[i, i] = Y_{dL}$ (in siemens) for different incident waves. In the first case (dashed lines), the lines are only connected to the ground—they are not interconnected, whereas in the second case (solid lines), nearest neighbors are interconnected by the resistors obtained from the admittance matrix $[Y_C]$. It is clearly seen that the minimum of the reflected power is achieved at values of Y_{dL}, which are dependent on the nature of the incident wave.

Table I shows a comparison between (1), (2), and (10) with respect to the entries of the termination network and the associated reflected powers. The sum—not the reflected power—given in (1) is minimized, with a minimum of 0.006, when the termination network is chosen according to the second column of Table I. However, the reflected power obtained from this network depends on the excitation. If a random excitation is assumed, a minimum of 0.107% reflection is achieved (column 2). This is comparable to the value obtained from direct minimization of the reflected power under the same excitation (column 4). When only line one is excited, the direct minimization of the reflected power according to (10) leads to practically zero reflected power, whereas (1) gives 0.14%.

If the termination network is determined from (2), a minimum reflected power of 1.56% is achieved with the resistors in the third column of Table I. It is interesting to note that the minimum is reached with unequal values of R_{12} and R_{45} on the one hand, and R_{23} and R_{34} on the other hand. For reference, the termination network obtained from the characteristic admittance Y_C is given in column five.

[1] *IMSL User's Manual*, IMSL Math/Library, Version 1.1, p. 802, Dec. 1989.

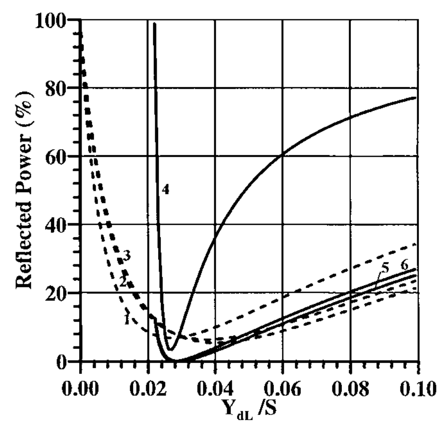

Fig. 2. Reflected power from a termination network for different form of incident excitations as a function of Y_{dL} (in siemens). Dashed lines correspond to a network with resistors connecting the lines to the ground only (1: line one excited; 2: line three excited; 3: line two excited). Solid lines correspond to a network with resistors connecting adjacent lines as well as the ground plane (4: all lines excited; 5: line two excited; 6: line three excited).

When the total reflected power is minimized, the optimal termination network depends on the incident wave. Assuming a random excitation, the minimum reflected power is 0.102% of the incident power. The corresponding network is that of column four in Table I. This minimum is of the same order as the reflected power, resulting from when the termination is that given by the characteristic admittance Y_C (0.125%) where the lines are connected to the ground plane and only nearest neighbors are interconnected (column five).

When only one line is excited, the reflected power is minimized when the excited line is terminated in a resistance approximately equal to that obtained from Y_C, with the other lines disconnected from the ground plane.

TABLE II
TERMINATION NETWORK FOR SEVEN ASYMMETRIC LINES

	equ. 1	equ. 2	equation (10)		
			line 1 excited	line 4 excited	random excitation
R_{11}	88.57	79.77	86.52	0.0069	87.54
R_{12}	178.40	38.54	175.17	175.17	163.55
R_{22}	219.49	180.03	0.01	0.06	240.12
R_{23}	124.02	90.92	114.54	114.54	101.72
R_{33}	139.36	139.40	0.05	0.17	146.48
R_{34}	122.77	49.84	82.07	82.07	97.44
R_{44}	179.26	144.12	0.10	928.14	241.23
R_{45}	99.09	51.38	75.06	75.06	80.87
R_{55}	145.75	129.20	0.10	0.16	152.75
R_{56}	178.22	90.60	177.94	177.94	147.08
R_{66}	343.87	346.66	0.05	0.07	390.29
R_{67}	68.43	69.46	63.92	63.92	62.63
R_{77}	82.88	71.42	0.05	0.45	81.57
$Pr\%$	1.92	6.82	2.10^{-5}	6.10^{-6}	1.61

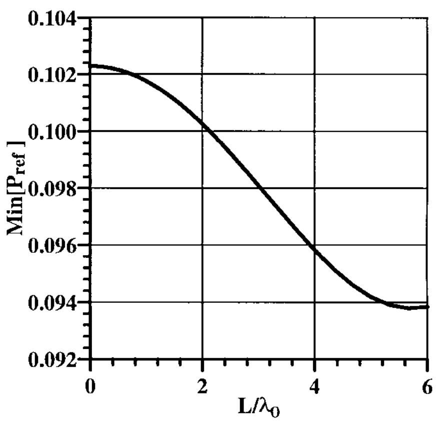

Fig. 3. Minimum reflected power as a function of L/λ_0 when a random voltage is launched on five symmetric lines at a distance L from the termination network.

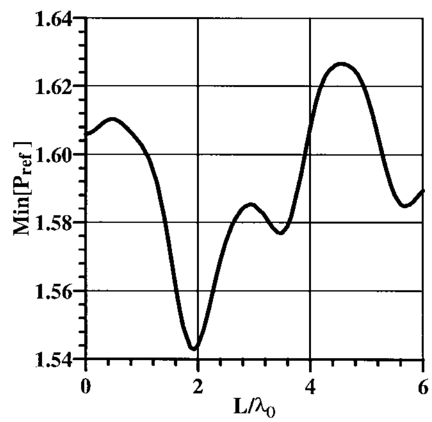

Fig. 4. Minimum reflected power as a function of L/λ_0 when a random voltage is launched on seven asymmetric lines at a distance L from the termination network.

The effect of the length of the transmission section on the termination network was also investigated. For a random-incident excitation at the generator, Fig. 3 shows the minimum total reflected power as a function of L/λ_0 (λ_0 is the free-space wavelength). The curve is periodic only if the modal propagation constants are commensurate, i.e., their ratios are rational numbers.

B. Seven Asymmetric Lines

We finally consider a system of seven asymmetric coupled lines and their termination networks. The dimensions of the lines are $w_1 = 0.6$ mm, $w_2 = 0.4$ mm, $w_3 = 0.45$ mm, $w_4 = 0.5$ mm, $w_5 = 0.5$ mm, $w_6 = 0.35$ mm, and $w_7 = 0.3$ mm. The substrate is 1-mm thick with $\epsilon_r = 12.9$. The line separations are $s_{12} = 0.2$ mm, $s_{23} = 0.1$ mm, $s_{34} = 0.2$ mm, $s_{45} = 0.1$ mm, $s_{56} = 0.2$ mm, and $s_{67} = 0.1$ mm. At 1 GHz, the effective dielectric constants ($\epsilon_{\rm eff} = (\beta_k/k_0)^2$, where k_0 is the free-space wave number) of the seven modes are found to be 10.1521, 8.0913, 7.3149, 7.0151, 6.9742, 6.9553, and 6.9512, respectively.

The termination network obtained from (1) comprises the resistors of the second column of Table II. The reflected power from this termination network under a random excitation is 1.92%, which is slightly higher than the value obtained from direct minimization of the reflected power (1.61%, column six). When only line one is excited, the same network reflects 0.72% of the incident powers, whereas direct minimization leads to less than 10^{-4}% under the same excitation. Under a random excitation, the network obtained from (2) leads to a minimum reflected power of 6.82% (column three).

If the total reflected power is minimized, the resulting network depends on the nature of the excitation. The results are summarized in columns four through six of Table II. Similar to the case of five symmetric lines, the lines are connected to the ground plane and only to their immediate neighbors. When only mode 1 is excited, the total reflected power is 4.8%, when the termination is obtained from the characteristic matrix Y_C (not shown in Table II). This minimum drops to 2.10^{-5}% when the termination network is optimized with respect to the resistors connecting the lines to the ground plane while keeping the interconnections between neighbors equal to $R_{1,j} = -1/[Y_C]_{i,j}$, $|i - j| = 1$.

Note that the different propagation constants and interactions between lines are fully taken into account. Even if a resistance of the termination network is optimized to be close to zero (e.g., columns four and five of Table II), it means that for the given excitation, a short needs to be placed between the respective line and ground. This

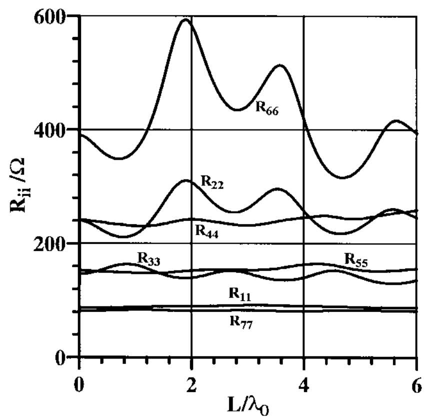

Fig. 5. Diagonal elements R_{ii} of optimal termination network under random excitation as a function of L/λ_0.

can be viewed as if the shorted line is part of the matching network required to minimize crosstalk between the remaining lines. However, on the other hand, if the shorted line is now excited, the termination is bound to fail. This fact supports our proposal for a termination network designed for random excitations.

The dependence of the total reflected power on the length of the transmission section, when a random voltage is incident at the generator, is shown in Fig. 4. The curve is periodic only if the modal propagation constants are commensurate. As an example of the dependence of the individual elements of the termination network on the line length, Fig. 5 shows the required variation of R_{11}, R_{22}, R_{33}, R_{44}, R_{55}, R_{66}, and R_{77} of the asymmetric seven-line structure in order to maintain optimum performance. It is interesting to note that the load element R_{66} is much more sensitive to the line length than the other resistances. Since all the excitations are assumed having equal probability, this effect is attributed to the fact that line number six is one of the narrowest (0.35 mm) with closest coupling distance (0.1 mm) to its immediate neighbors. Line number seven, although the narrowest, is immediately coupled only to line number six, and does not show as strong a sensitivity.

V. Conclusions

An analysis of the termination networks which lead to maximum power delivery, or minimum total reflected voltage, is presented for tightly coupled microstrip lines. Since an ideally matched matrix-type termination network can not be constructed, a termination network which minimizes the reflected power is designed instead. Even when the individual entries of the reflection matrix $[\rho_V]_{ij}$ are not small, it is possible to considerably reduce the reflected power or voltage by adjusting the phases and magnitudes of the individual reflected waves. Since the termination network which insures minimum power reflection depends on the nature of the incident excitation, the terminology *adaptive termination network* is arguably more appropriate. For unknown excitation, which is usually the case in practice, a termination network based on random excitation is proposed.

References

[1] K. D. Marx, "Propagation modes, equivalent circuits, and characteristic terminations for multiconductor transmission lines with inhomogeneous dielectrics," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-21, pp. 450–457, July 1973.

[2] J. T. Kuo and C. K. C. Tzuang, "A termination scheme for high-speed pulse propagation on a system of tightly coupled coplanar strips," *IEEE Trans. Microwave Theory Tech.,* vol. 42, pp. 1008–1015, June 1994.

[3] G. T. Lei, G. W. Pan, and B. K. Gilbert, "Examination, clarification, and simplification of modal decoupling method for multiconductor transmission lines," *IEEE Trans. Microwave Theory Tech.,* vol. 43, pp. 2090–2100, Sept. 1995.

[4] T. Itoh, "Spectral domain immittance approach for dispersion characteristics of generalized printed transmission lines," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-28, pp. 733–736, July 1980.

[5] D. Mirshekar-Syahkal, *Spectral Domain Method for Microwave Integrated Circuits.* New York: Wiley, 1990.

[6] S. Amari and J. Bornemann, "Non-perturbative fullwave analysis of lossy planar circuits," in *IEEE MTT-S Int. Microwave Symp. Dig.,* Orlando, FL, May 1995, pp. 1277–1280.

An FD–FD Formulation for the Analysis of the Optical Axis Misalignment Effect on Propagation Characteristics of Anisotropic Dielectric Waveguides

Carlos Leônidas da S. S. Sobrinho and Attílio J. Giarola

***Abstract*—A finite-difference frequency-domain (FD–FD) formulation is developed to study the dispersion characteristics of anisotropic dielectric waveguides with their optical axes not aligned with the coordinate-system axes. In this analysis, the optical axes are initially assumed to be aligned with the coordinate-system axes such that the electric-permittivity and magnetic-permeability tensors are diagonal. The optical axes of the anisotropic dielectric are then rotated an angle θ (or ϕ) with respect to the coordinate-system axes. While the FD–FD formulation developed is general, it is applied here only to waveguides containing uniaxial anisotropic dielectrics. The results show that accurate optical-axis orientation is important in the design of dielectric waveguides.**

***Index Terms*—Optical axial misalignment.**

I. Introduction

Various methods have been used in the analysis of the propagation characteristics of dielectric waveguides, with applications in integrated circuits in the millimeter and optical frequency bands. Among these methods, the following are of interest:

1) the finite-difference frequency-domain (FD–FD) formulation [1], [2];
2) the two-dimensional (2-D) finite-difference time-domain (2-D FDTD) method [3];
3) the finite-element method (FEM) [4];
4) the transmission-line method (TLM) [5].

While the FD–FD, FEM, and TLM in the frequency-domain methods are equivalent, this is not true for the FD–FD and 2-D

Manuscript received December 12, 1995; revised June 20, 1997. This work was supported in part by the Brazilian agencies CNPq, TELEBRÁS, FINEP, and CAPES.

C. L. da S. S. Sobrinho is with the Electrical Engineering Department, Federal University of Pará (UFPa), Belém, PA, Brazil 66075-900.

A. J. Giarola is with the School of Electrical Engineering, State University of Campinas (UNICAMP), Campinas, SP, Brazil, 13083-970.

Publisher Item Identifier S 0018-9480(97)07118-4.